

**Lesson Overview**

One common use of modeling is the creation of a single numerical rating that incorporates multiple measures for the purposes of ranking products or people. This activity uses the measures commonly collected for football quarterbacks and provides an opportunity for students to create their own quarterback rating models. These can be compared to the formula in actual use by the National Football League (NFL) and to other formulas, such as that used by the National Collegiate Athletic Association (NCAA).

**About the Lesson and Possible Course Connections:**

The activity can be used with introductory algebra students, and lends itself to a group project assignment. There are nice connections to averages and ratios as a tool for analyzing information, in particular for accounting for the difference in the units associated with the data. The rating formula is a good example of a multivariable function, and its structure is easily analyzed. The activity could also be adapted for middle school students by using a smaller subset of the list of players.

**Learning Goals**

Students will be able to:

1. model a contextual situation mathematically and use the model to answer a question.
2. represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
3. find and interpret linear equations to model relationships between two quantitative variables.
4. use proportional relationships to solve real-world and mathematical problems.

**CCSS Standards****Algebra Standards:**

- A-SSE.B, A-SSE.B.1
- A-SSE.A, A-SSE.A.1
- A-CED.A, A-CED.A.2
- A-SSE.A.3

**Interpreting Data Standards:**

- S-ID.B, S-ID.B.6

**Ratio and Proportional Relationships Standards:**

- 7.RP.A, 7.RP.A.1

**Mathematical Practice Standards**

- SMP.4

### Lesson Materials

- Compatible TI Technologies:



TI-Nspire CX Handhelds,



TI-Nspire Apps for iPad®,



TI-Nspire Software

- Modeling QB Passer Rating\_Student.pdf
- Modeling QB Passer Rating\_Student.doc
- Modeling QB Passer Rating.tns
- Modeling QB Passing Rating\_Teacher Notes

### Background

Who is the GOAT (greatest of all time) among past and current NFL quarterbacks? Many passing statistics are gathered for quarterbacks, including numbers of passing attempts, completed passes, yards gained, touchdown passes, and on the negative side, interceptions. How can all this information be combined in a way that allows us to give each quarterback a single numerical rating that could be used to compare and rank different quarterbacks? Students will use the data provided in the Appendix or in the calculator file, which can be provided directly to them containing the data.

**Teacher Tip:** Students should open the .tns file with the data. If they enter the data by hand, they are likely to make errors. Younger students might work with a subset, for example the current players. Students do not need a background in regression or multiple regression; they can think about different ways to combine the numbers in order to create a model for rating each quarterback based on the data. If they do have a background in regression, encourage them to think about other alternatives as well. Students do not really have to have a background in football (although it might make the task more interesting). Knowing that they are looking for a way to rank the players from best down, they can just play with the numbers. They may choose to use part or all of the information but should have some reason for their decision.

**Facilitating the Lesson**

Introduction: Present the class with the data – either projected or paper copies (See Appendix). Begin the discussion by having each student write down the answers to the following questions.

1. What do you notice? (Some made a lot of touchdowns and some not so many; Aaron Rodgers has very few interceptions; Tom Brady, Drew Brees, Brett Favre and Peyton Manning made more yards passing compared to all of the others; Favre had a lot of interceptions compared to the others, ...)
2. What do you wonder about? (If the interceptions balance out the touchdowns, does that indicate a “mediocre” quarterback? Are the quarterbacks with the most touchdowns really the best?)
3. Which of the data do you think most important? Least important? Are the number of attempts important? Why or why not?

Discuss the answers with the class, recording the things students notice and wonder on the board or chart paper. After the class has responded, if someone wonders who the best quarterback is, tell the class that is what you are going to investigate. If it does not occur in the wonderings, point out that sportswriters and others are wondering about who is best, so you are going to investigate. (Save the other wonderings as possible further investigations).

There are several ways this lesson can be implemented in the classroom:

**1) Open-Ended Approach:**

First, ask the students to think alone for a few minutes and then write down how they would start to rank the quarterbacks. Then, put them in pairs or threes for further discussion.

Student Instructions: In your groups,

- Exchange your ideas.
- Decide as a group how you will begin to analyze the data. Give each member of the group a job to do that will help you in the work.
- Decide whether your approach seems reasonable for the data. Explain why you think your model is good for rating the quarterbacks. What are the drawbacks, if any, to your model?

**2) More-Structured Approach to Finding a Model:**

Put students into groups to find a method to rate the players. Remind them that they will be expected to defend their process for rating the players. Pose the following questions to help push their thinking:

- Will a graph help?
- What relationships might you explore graphically to help determine a formula?
- Are all of the categories equally important? Why or why not?
- How can you account for a quarterback who played for many years as opposed to one who played for much less time?
- Can you create a formula?

## What to Expect: Example Student Solutions

### EX 1: Graphing the Data

Some students might consider graphing the data. For example, if a scatter plot of (completions, touchdowns) is graphed, the relationship between completions and touchdowns seems to be linear (figure 1). Help them consider all of the data: for example, what happens with a quarterback who has high numbers of both completions and touchdowns, but also has a lot of interceptions (figure 2)?

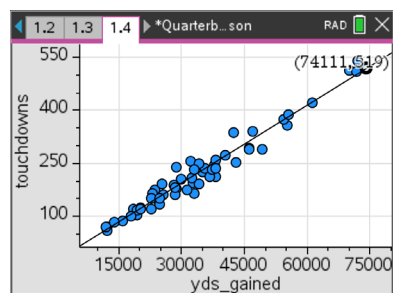


Figure 1: Touchdowns vs. Yards Gained

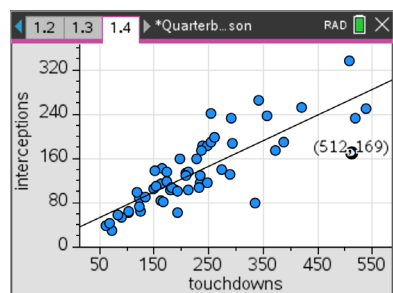


Figure 2: Interceptions vs. Touchdowns

Students may investigate why it might be useful to divide by the number of attempts and should explain why the outcomes are different (Figures 3 and 4).

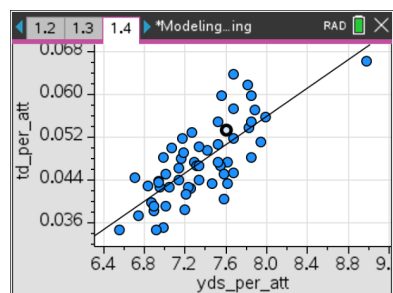


Figure 3: Touchdowns vs Yards Gained but now accounting for attempts

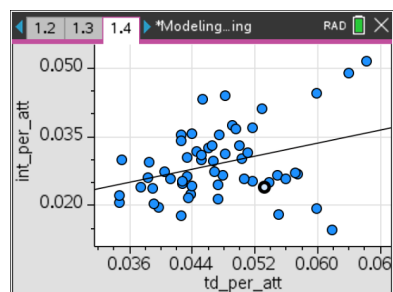


Figure 4: Interceptions vs. Yards Gained but now accounting for attempts

**EX #2: Creating a Formula**

Some students might create a formula; they should recognize that using averages (rates) per attempt (for example, yards per attempt, completions per attempt, etc.) rather than counts might be useful for comparing two players with very different numbers of attempts – dividing by the number of attempts can accomplish this. One possible such model might be the following, where the interception rate is subtracted from the others.

$$\frac{C}{A} + \frac{T}{A} + \frac{Y}{A} - \frac{I}{A} = R$$

Notice that  $Y$  (measured in yards) is much larger than the other values, so this simple sum of averages makes the contribution of yards per attempt to the rating  $R$  much greater than any of the other averages.

To account for this, different coefficients (weights) for the terms could be introduced to adjust for the different sizes of these averages, as well reflecting some subjective judgment of their relative importance. For example, one such weighting might be:

$$5\frac{C}{A} + 10\frac{T}{A} + 2\frac{Y}{A} - 10\frac{I}{A} = R$$

Some students might look for a way to compare each player's performance "score" in a category (such as touchdowns per attempt) to the best "score" achieved among all the players. For example, Graham's touchdowns per attempt is the best of all players at 0.066, so each player's touchdowns per attempt could be divided by 0.066 to get a statistic relative to Graham's best performance, which would be 1.0 (from  $0.066/0.066$ ). Brees' touchdowns per attempt average of 0.053 would be converted to the value  $0.053/0.066 = 0.80$  or 80% of the best performance in the data. A similar benchmark for each data category could be computed, and the other players' scores converted to this new scale. These converted scores, now on the same scale could be combined to create an overall rating for each player.



### Validating the Models

*Students should validate their models either by asking whether the models make sense in different scenarios related to the context or by finding other information to reflect against the model. One strategy might be to have each group consider whether the models created by other groups make sense.*

*Some questions are suggested below that might be useful in helping students think about whether their model was reasonable:*

- How do the rankings of the quarterbacks by different groups in class compare to each other? What might explain any big differences?
- Would different graphical representation generate different rankings that are reasonable? Why or why not?
- What is the highest rating someone could have from the model? The lowest? Do these extremes make sense?
- Popularly, several quarterbacks have been recognized as being “great” (e.g., Aaron Rodgers, Drew Brees, Tom Brady, ...) Does the model place them appropriately? What might need to be changed in the model to take the statistics for these players into account?
- One of the “great” quarterbacks from many years ago was Johnny Unitas, who played with the Baltimore Colts from 1956 to 1973. He had 5186 attempts, made 2830 completions, 290 touchdowns and 253 interceptions. The total number of yards gained in passing was 39,768. Would the model place him in a rank that made sense? Why did he not earn a place in this set of top quarterbacks?
- One of the up and coming quarterbacks is Patrick Mahomes, of the Kansas City Chiefs with 724 completions, 1099 attempts, 9,412 yards gained, 76 touchdowns and 18 interceptions after the 2019 regular season. Would the model place him in a sense making rank?



### Extension

Consider the actual formula\* used by the NFL to rate quarterbacks described below, where :

A = attempts                      T = touchdowns                      Y = yards gained  
C = completions                      I = interceptions

**Method 1.** The NFL quarterback rating  $R$  is given by this formula:

$$R = \frac{50 + 2,000 \frac{C}{A} + 8,000 \frac{T}{A} - 10,000 \frac{I}{A} + 100 \frac{Y}{A}}{24}$$

### Discussion Questions:

1. What are some interesting questions you might pose about this formula?
2. An alternate approach is described below as Method 2. How do the two approaches compare?
3. Find the ratings for the quarterbacks in the table using either the formula or the method 2, described below. How did your model compare to the one used by the NFL?

**Method 2.** Here is an alternative method of computing the NFL quarterback rating  $R$ :

- Step 1: Complete passes divided by pass attempts. Subtract 0.3, then divide by 0.2
- Step 2: Passing yards divided by pass attempts. Subtract 3, then divide by 4.
- Step 3: Touchdown passes divided by pass attempts, then divide by .05.
- Step 4: Start with .095, and subtract interceptions divided by attempts. Divide the product by .04.

The sum of each step cannot be greater than 2.375 or less than zero. Add the sum of Steps 1 through 4, multiply by 100, and divide by 6.

### Discussion Question:

4. Are the two methods really equivalent? Why or why not?

\* [https://en.wikipedia.org/wiki/Passer\\_rating](https://en.wikipedia.org/wiki/Passer_rating)

5. College quarterbacks are rated using a different formula:

$$R_{NCAA} = \frac{8.4 Y + 330 TD + 100 C - 220 I}{A}$$

How would the ranking from this formula compare to your model?  
What about to the NFL method?

6. Choose a model (yours, the NFL or the College Ranking) to rate the quarterbacks in your school's league. How well do you think the ranking works?

### Appendix: Data Set for Lesson

The table below shows the results of using the NFL quarterback rating formula (either Method) for these quarterbacks. In the student handout there is a copy of this table without the rating column.

NFL Ratings based on the NFL Quarterback ranking formula (Method 1) given earlier:

|                    | TDs | Pass completion | Yds gained | Attempts | Interceptions | Rating |
|--------------------|-----|-----------------|------------|----------|---------------|--------|
| Troy Aikman        | 165 | 2898            | 32942      | 4715     | 141           | 81.6   |
| Kent Anderson      | 197 | 2654            | 32838      | 4475     | 160           | 81.9   |
| *Tom Brady         | 512 | 5967            | 70138      | 9318     | 169           | 97.6   |
| *Drew Brees        | 519 | 6559            | 74111      | 9744     | 233           | 97.8   |
| *Sam Bradford      | 103 | 1855            | 19449      | 2967     | 61            | 84.5   |
| Marc Bulger        | 122 | 1969            | 22814      | 3171     | 93            | 84.4   |
| *Derek Carr        | 122 | 1716            | 18387      | 2741     | 87            | 89.2   |
| Jay Cutler         | 227 | 3048            | 35133      | 4920     | 160           | 85.5   |
| Kurt Cousins       | 125 | 1756            | 20119      | 2641     | 65            | 94.8   |
| Dante Culpepper    | 149 | 2016            | 24153      | 3199     | 106           | 87.8   |
| Randall Cunningham | 207 | 2429            | 29979      | 4289     | 134           | 81.5   |
| *Andy Dalton       | 188 | 2443            | 28100      | 3921     | 104           | 88.8   |
| Len Dawson         | 239 | 2136            | 28711      | 3741     | 183           | 82.6   |
| Boomer Esiason     | 247 | 2969            | 37920      | 5205     | 184           | 81.1   |
| Brett Favre        | 508 | 6300            | 71838      | 10169    | 336           | 86.    |
| *Joe Flacco        | 212 | 3499            | 38245      | 5670     | 136           | 84.1   |
| Dan Fouts          | 254 | 3297            | 43040      | 5604     | 242           | 80.2   |
| Rich Gannon        | 180 | 2533            | 28743      | 4206     | 104           | 84.7   |





# Modeling Quarterback Passer Rating

## TEACHER NOTES

|                  |     |      |       |      |     |      |
|------------------|-----|------|-------|------|-----|------|
| Jeff Garcia      | 161 | 2264 | 25537 | 3676 | 83  | 87.5 |
| Otto Graham      | 174 | 1464 | 23584 | 2626 | 135 | 86.6 |
| Trent Green      | 162 | 2266 | 28475 | 3740 | 114 | 86   |
| David Garrard    | 89  | 1406 | 16003 | 2281 | 54  | 85.8 |
| Brian Griese     | 119 | 1752 | 19440 | 2794 | 99  | 82.7 |
| Matt Hasselbeck  | 212 | 3222 | 36638 | 5330 | 104 | 82.4 |
| Sonny Jurgensen  | 255 | 2433 | 32224 | 4262 | 189 | 82.6 |
| Colin Kaepernick | 72  | 1011 | 12271 | 1692 | 30  | 88.9 |
| Dave Krieg       | 261 | 3105 | 38147 | 5311 | 199 | 81.5 |
| *Casey Keenum    | 61  | 1090 | 12167 | 1759 | 39  | 85.5 |
| Jim Kelly        | 237 | 2874 | 35467 | 4779 | 175 | 84.4 |
| *Andrew Luck     | 166 | 1945 | 23029 | 3208 | 81  | 89.3 |
| * Eli Manning    | 357 | 4755 | 55371 | 7898 | 237 | 84.2 |
| Peyton Manning   | 539 | 6125 | 71940 | 9380 | 251 | 96.5 |
| Dan Marino       | 420 | 4967 | 61361 | 8358 | 252 | 86.4 |
| *Marcus Mariota  | 69  | 1005 | 11894 | 1592 | 42  | 89.5 |
| Donovan McNabb   | 234 | 3170 | 37276 | 5374 | 117 | 85.6 |
| Steve McNair     | 174 | 2733 | 31304 | 4544 | 119 | 82.8 |
| Joe Montana      | 273 | 3409 | 40551 | 5391 | 139 | 92.3 |
| Warren Moon      | 291 | 3988 | 49325 | 6823 | 233 | 80.9 |
| *Cam Newton      | 182 | 2321 | 28469 | 3891 | 107 | 86.7 |
| Carson Palmer    | 294 | 3941 | 46247 | 6307 | 187 | 87.9 |
| Chad Pennington  | 102 | 1632 | 17823 | 2471 | 64  | 90.1 |
| *Matt Ryan       | 290 | 4006 | 46103 | 6131 | 132 | 94.6 |



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|                      |     |      |       |      |     |       |
|----------------------|-----|------|-------|------|-----|-------|
| * Philip Rivers      | 373 | 4481 | 54299 | 6939 | 174 | 96    |
| *Aaron Rodgers       | 336 | 3520 | 42476 | 5432 | 80  | 103.1 |
| * Ben Roethlisberger | 388 | 4552 | 55527 | 7073 | 189 | 94.1  |
| Tony Romo            | 248 | 2829 | 34183 | 4335 | 117 | 97.1  |
| *Matt Schaub         | 133 | 2098 | 24887 | 3281 | 90  | 89.1  |
| *Alex Smith          | 193 | 3082 | 34068 | 4941 | 101 | 87.3  |
| Roger Staubach       | 153 | 1685 | 22700 | 2958 | 109 | 83.4  |
| *Matthew Stafford    | 235 | 3334 | 38144 | 5341 | 129 | 88.4  |
| Bart Starr           | 152 | 1808 | 24718 | 3149 | 138 | 80.5  |
| Ryan Tannehill       | 122 | 1796 | 20141 | 2858 | 72  | 87.8  |
| Fran Tarkenton       | 342 | 3686 | 47003 | 6467 | 266 | 80.4  |
| Kurt Warner          | 208 | 2666 | 32344 | 4070 | 128 | 93.7  |
| *Russell Wilson      | 192 | 2065 | 25201 | 3211 | 62  | 100.3 |
| *Jameis Winston      | 83  | 1127 | 13947 | 1839 | 57  | 87.3  |
| Steve Young          | 232 | 2667 | 33124 | 4149 | 107 | 96.8  |

\* Active in 2018

### Resources

[https://en.wikipedia.org/wiki/List\\_of\\_National\\_Football\\_League\\_career\\_passer\\_rating\\_leaders](https://en.wikipedia.org/wiki/List_of_National_Football_League_career_passer_rating_leaders)  
career stats at nfl.com

Data can also be found by searching for each individual player's individual statistics.